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SOME RESULTS ON P-SKEW-BINORMAL OPERATOR IN MINKOWSKI SPACE M

D. Krishnaswamy, J. Prabha

[1] Department of Mathematics, Annamalai University, Annamalai Nagar, Tamil Nadu,
(E-mail: krishna_swamy2004@yahoo.co.in)

ORCID:0000-0002-1192-8520

[2] Department of Mathematics, AnnamalaiUniversity, Annamalai Nagar, Tamil Nadu,
(E-mail: krithika0392@gmail.com)

ORCID:009-0005-6992-9226

Abstract

In this paper, we investigate Skew-bi-normal operator in Minkowski space M. Furthermore, we study some properties of Skew- bi-normal operator in Minkowski space M. Also the relation between bi-normal and Skew-bi-normal are discussed.

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1. INTRODUCTION

Throughout we shall deal with $C^{n\times n}$, the space of $n \times n$ complex matrices. Let C^n be the space of complex n-tuples, we shall index the components of a complex vector in C^n from 0 to n-1, that is $u = (u_0, u_1, u_2, \dots, u_{n-1})$. Let G be the Minkowski metric tensor defined by $Gu = (u_0, -u_1, -u_2, \dots, -u_{n-1})$. Clearly the Minkowski metrix matrix

$$G=\begin{pmatrix} 1 & 0 \\ 0 & -I_{n-1} \end{pmatrix} \tag{1}$$

$G = G^*$ and $G^2 = I_n$. In [[13]], Minkowski inner product on C^n is defined by $(u, v) = [u, Gv]$, where $[.,.]$ denotes the conventional Hilbert space inner product. A space with Minkowski inner product is called the Minkowski space and denoted as M. For $A \in C^{n\times n}$, $x, y \in C^n$ by using (1),

$(Ax, y) = [Ax, Gy] = [x, A^*Gy]$

$= [x, G(GA^*G)y] = [x, GA^\sim y] = (x, A^\sim y)$.

Where $A^\sim = GA^*G$. The matrix $A^\sim$ is called the Minkowski adjoint of A in M. Naturally, we call a matrix $A \in C^{n\times n}$ m-symmetric in M if $A = A^\sim$. For $A \in C^{n\times n}$, let A^* , $A^\sim$, A^{m} , $A^\dagger$, $R(A)$ and $N(A)$ denote the conjugate transpose, Minkowski adjoint, Minkowski inverse, Moore-Penrose inverse, range space and null space of a matrix A respectively. I_n denote the identity matrix of order $n \times n$.

Generalized inverses of matrices have important roles in theoretical and numerical methods of linear algebra. The most significant fact is that we can use generalized inverse of matrices, in the case when ordinary inverses do not exists, in order to solve some matrix equations. Similar reasoning can be applied to linear (bounded or unbounded) operators on Banach and Hilbert spaces.

Different authors [11], [14] and [9] have generalized the fundamental properties of various operators. Furthermore, we have explored various characteristic of the Skew-bi-normal operator. Ultimately, we established the connection between m-symmetric and Skew-bi-normal operators.

2. Preliminaries

Definition 2.1. An operator E is said to be normal in M *if* $E^\sim E = EE^\sim$

Definition 2.2. An operator E is said to be bi-normal in M if $E^\sim EEE^\sim = EE^\sim E^\sim E$

Definition 2.3. An operator E is said to be skew-normal in M if $(EE^\sim)E = E(E^\sim E)$

Definition 2.4. An operator E is said to be skew-binormal in M if $(E^\sim EEE^\sim)$

$$E = E(EE^\sim E^\sim E).$$

Definition 2.5. Let E be a bounded linear operator on minkowski space M.
Then E is said to be P-skew-bi-normal opertor in minkowspace M if and only if $(E^\sim EEE^\sim)(E^\sim)^p = (E^\sim)^p(EE^\sim E^\sim E)$, where p is a nonnegative integer.

1. 3. Skew-binormal operator in Minkowski space M

An operator E is skew-binormal operator in Minkowski space M if $(E^\sim EEE^\sim)$

$E = E(EE^{\sim}E^{\sim}E)$.

Theorem 3.1. If E is m-symmetrix and skew-binormal operator in minkowski space M then the following holds:

a. Suppose β is any scalar then βE is also skewnormal operator in M.

b. The restriction E/B of E to any closed subspace B of M that reduces E.

Proof:

(i) Given, E is skew-binormal operator in M,

Since, $(E^{\sim}EEE^{\sim})E = E(EE^{\sim}E^{\sim}E)$,If β is any real scalar,

then $(\beta E)^{\sim} = G(\beta E)^{*}G = \beta GE^{*}G = \beta E^{\sim}$ [since $\beta^{*} = \beta$] $[(\beta E)^{\sim}(\beta E)(\beta E)(\beta E)^{\sim}](\beta E) = \beta E^{\sim}\beta E\beta E\beta E^{\sim}\beta E = \beta^{5}E^{\sim 2}E^{3}$ ……………………………………………………………………………(1)

$(\beta E)[(\beta E)(\beta E)^{\sim}(\beta E)^{\sim}(\beta E)] = \beta E\beta E\beta E^{\sim}\beta E^{\sim}\beta E = \beta^{5}E^{\sim 2}E^{3}$ ………………………………(2)

From (1) and (2), βE is also skew-binormal operator in M

(ii) If E is skew-binormal in M then $(E^{\sim}EEE^{\sim})E = E(EE^{\sim}E^{\sim}E)$ Consider,

$$[(E/B)^{\sim}(E/B)(E/B)(E/B)^{\sim}](E/B) = [E^{\sim}EEE^{\sim}/B](E/B)$$

$$= [E^{\sim}EEE^{\sim}]E/B]$$

$$= E[EE^{\sim}E^{\sim}E]/B \text{ [Since E is skew-binormal]}$$

$$= (E/B)[(E/B)(E^{\sim}/B)(E^{\sim}/B)(E/B)]$$

Hence E/B is also skew-binormal operator in M.

Theorem 3.2. If E is m-symmetrix and normal operator in Minkowski space M then E is skew-binormal operator in minkowki space

Proof:

If E is normal operator in Minkowski space M then $E^{\sim}E=EE^{\sim}$

$$(E^{\sim}EEE^{\sim})E = E^{\sim}EEE^{\sim}E$$

$$= EE^{\sim}EE^{\sim}E$$

$$= EE^{\sim}EE^{\sim}E$$

$$= EEE^{\sim}E^{\sim}E$$

$$= E(EE^{\sim}E^{\sim}E)$$

$$(E^{\sim}EEE^{\sim})E = E(EE^{\sim}E^{\sim}E)$$

Hence E is skew-binormal operator in Minkowski space M.

Theorem 3.3. If E is m-symmetrix skew- binormal operator in Minkowski space M and F is skew-binormal operator in Minkowski space M . If E and F are doulbly commuting then EF is skew-binormal operator in Minkowski space M.

Proof:

Given E and F are skew- binormal operator in Minkowski space M and by using definition of skew-binormal operator we know that,

$$(E^{\sim}EEE^{\sim})E = E(EE^{\sim}E^{\sim}E)$$

and $(F^{\sim}FFF^{\sim})F = F(FF^{\sim}F^{\sim}F)$ and given m-symmestrix $E^{\sim}F=FE^{\sim}$

$$[(FE)^{\sim}(FE)(FE)(FE)^{\sim}](FE) = (FE)^{\sim}(FE)(FE)(FE)^{\sim}(FE)$$

$$= E^{\sim}F^{\sim}FEFEE^{\sim}F^{\sim}FE$$

$$= F^{\sim}E^{\sim}FEEFE^{\sim}F^{\sim}EF$$

$$= F^{\sim}FE^{\sim}EEE^{\sim}FEF^{\sim}F$$

$$= F^{\sim}FE^{\sim}EEE^{\sim}EFF^{\sim}F$$

$$= F^{\sim}F(E^{\sim}EEE^{\sim})EFF^{\sim}F$$

$$= F^{\sim}F(EE^{\sim}E^{\sim}E)EFF^{\sim}F$$

$$= F^{\sim}FEEE^{\sim}E^{\sim}EFF^{\sim}F \text{ [Since E is skew-binormal]}$$

$$= F^{\sim}EFEE^{\sim}E^{\sim}FEF^{\sim}F$$

$$= EF^{\sim}EFE^{\sim}FE^{\sim}F^{\sim}EF$$

$$= EEF^{\sim}FE^{\sim}FF^{\sim}E^{\sim}FE$$

$$= EEF^{\sim}E^{\sim}FFF^{\sim}FE^{\sim}E$$

$$= EEE^{\sim}F^{\sim}FFF^{\sim}FE^{\sim}E$$

$$= EEE^{\sim}(F^{\sim}FFF^{\sim})FE^{\sim}E$$

$$= EEE^{\sim}F(F^{\sim}FFF^{\sim})E^{\sim}E \text{ [Since F is skew-binormal]}$$

$$= EEE^{\sim}FF^{\sim}FFF^{\sim}E^{\sim}E$$

$$= EEE^{\sim}FF^{\sim}FF^{\sim}FE^{\sim}E$$

$$= EEFE^{\sim}FF^{\sim}F^{\sim}E^{\sim}FE$$

$$= EFEE^{\sim}FF^{\sim}F^{\sim}E^{\sim}FE$$

$$= EFEFE^{\sim}F^{\sim}E^{\sim}F^{\sim}FE$$

$$= (FE)(FE)(FE)^{\sim}(FE)^{\sim}(FE)$$

$$= (FE)(FE)[(FE)^{\sim}(FE)^{\sim}(FE)]$$

Hence FE is skew-binormal operator in Minkowski space M.

Theorem 3.4. Let, $E = U|E|$ be the polar decomposition of an operator E in M and U is unitary operator.. Then, $E = U|E|$ is skew-binormal in M if $U|E| = |E|U$, $U^{\sim}|E| = |E|U^{\sim}$.

Proof:

Consider,

$$(E^{\sim}EEE^{\sim})E - E(EE^{\sim}E^{\sim}E)$$

$$= (|E|U^{\sim}U|E|U|E|E|U^{\sim})U|E| - U|E|(U|E||E|U^{\sim}|E|U^{\sim}U|E|)$$

$$= (|E|^2U|E|^2U^{\sim})U|E| - U|E|(|E|UU^{\sim}|E||E|^2)$$

$$= |E|^2U|E|^2|E| - U|E||E|^2|E|^2$$

$$= |E|^2|E|^2U|E| - U|E||E|^2|E|^2$$

$$= 0$$

Theorem 3.5. Let $E \in M$ be m-symmetrix skew-binormal operator which is unitarily equivalent to an operator E if and only if $EU = UE, EU^{\sim} = U^{\sim}E, E^{\sim}U = UE^{\sim}$, then F is skew- binormal operator in Minkowski space M.

Proof:

Given that E is unitarily equivalent to F, there is a unitary operator U such that $F = U^{\sim}EU$ which implies $F^{\sim} = U^{\sim}E^{\sim}U$.

We must show that $(F^{\sim}FFF^{\sim})F = F(FF^{\sim}F^{\sim}F)$.

Since E is skew-binormal then $(E^{\sim}EEE^{\sim})E = E(EE^{\sim}E^{\sim}E)$

Consider,

$$(F^{\sim}FFF^{\sim})F = (U^{\sim}E^{\sim}UU^{\sim}EUU^{\sim}EUU^{\sim}E^{\sim}U)U^{\sim}EU$$

$= U\tilde{}E\tilde{}UU\tilde{}EUU\tilde{}EUU\tilde{}E\tilde{}UU\tilde{}EU$

$= U\tilde{}E\tilde{}EEE\tilde{}EU$ [Since U is unitary operator]

$= U\tilde{}E\tilde{}EEE\tilde{}UE$

$= U\tilde{}E\tilde{}EEUE\tilde{}E$

$= U\tilde{}E\tilde{}EUEE\tilde{}E$

$= U\tilde{}E\tilde{}UEEE\tilde{}E$

$= U\tilde{}UE\tilde{}EEE\tilde{}E$

$= E\tilde{}EEE\tilde{}E$

$= (E\tilde{}EEE\tilde{})E$ .... (1)

$F(FF\tilde{}F\tilde{}F) = U\tilde{}EU(U\tilde{}EUU\tilde{}E\tilde{}UU\tilde{}E\tilde{}UU\tilde{}EU)$

$= U\tilde{}EUU\tilde{}EUU\tilde{}E\tilde{}UU\tilde{}E\tilde{}UU\tilde{}EU$

$= U\tilde{}EEE\tilde{}E\tilde{}EU$ [Since U is unitary operator]

$= U\tilde{}EEE\tilde{}E\tilde{}UE$

$= U\tilde{}EEE\tilde{}UE\tilde{}E$

$= U\tilde{}EEUE\tilde{}E\tilde{}E$

$= U\tilde{}UE\tilde{}EEE\tilde{}E$

$= EEE\tilde{}E\tilde{}E$

$= E(EE\tilde{}E\tilde{}E)$

$= (E\tilde{}EEE\tilde{})E$ ....(2)

From (1) and (2) therefore F is skew-binormal.

Theorem 3.6. If the operator E is m-symmetrix operator and also skew -binormal operator in Minkowski space M, then $E\tilde{}$ is also skew binormal operator in Minkowski space M .

Proof:

Since E is skew-binormal operator, we have

$(E\tilde{}EEE\tilde{})E = E(EE\tilde{}E\tilde{}E)$………..(1)

Since E is m-symmetric , $E\tilde{} = E$

$$(E^{\sim}EEE^{\sim})E = E^{\sim}EEE^{\sim}E$$

$$= EE^{\sim}E^{\sim}EE^{\sim}$$

$$= E^{2}(E^{\sim})^{3} \cdots\cdots\cdots\cdots (2)$$

$$E(EE^{\sim}E^{\sim}E) = EEE^{\sim}E^{\sim}E$$

$$= E^{\sim}E^{\sim}EEE^{\sim}$$

$$= E^{2}(E^{\sim})^{3} \cdots\cdots\cdots\cdots (3)$$

From (2) and (3), $E^{\sim}$ is skew-binormal operator in Minkowski space M .

Theorem 3.7. If E is m-symmetric, then E is skew-binormal operator in Minkowski space $\mathbf{M}$.

Proof:

Since E is m-symmetric operator, $E^{\sim} = E$

Now,

$$(E^{\sim}EEE^{\sim})E = E^{\sim}EEE^{\sim}E = EEEEE = E^{5} \cdots\cdots\cdots\cdots (1)$$

$$E(EE^{\sim}E^{\sim}E) = EEE^{\sim}E^{\sim}E = EEEEE = E^{5} \cdots\cdots\cdots\cdots (2)$$

From (1) and (2)

$$(E^{\sim}EEE^{\sim})E = E(EE^{\sim}E^{\sim}E)$$

Hence E is skew-binormal operator.

Theorem 3.8. If E is any operator on a Minkowski space in M. Then

$(E + E^{\sim})$ is skew-binormal in M.

$EE^{\sim}$ is skew-binormal in M.

$E^{\sim}E$ is skew-binormal in M.

$I + E^{\sim}E,\ I + EE^{\sim}$ are skew-binormal in M.

Proof:

(i) Let $W = E + E^{\sim}$

$$W^{\sim} = (E + E^{\sim})^{\sim}$$

$$= E + E^{\sim}$$

$$= W$$

Hence, W is a m-symmetric operator.

We know that every m-symmetric operator is skew-binormal. Therefore, $W = E + E^{\sim}$ is skew-binormal operator in M.

(ii) $(EE^{\sim})^{\sim} = EE^{\sim}$

Hence, $EE^{\sim}$ is m-symmetric operator, so $EE^{\sim}$ is skew-binormal operator in M.

(iii) $(E^{\sim}E)^{\sim} = E^{\sim}E$

Hence, $(E^{\sim}E)$ is m-symmetric operator, so $(E^{\sim}E)$ is skew-binormal operator in M.

(iv) $(I + E^{\sim}E)^{\sim} = I + E^{\sim}E$

$(I + EE^{\sim})^{\sim} = I + EE^{\sim}$

Hence, $I + E^{\sim}E, I + EE^{\sim}$ are m-symmetric operator, so $I + E^{\sim}E, I + EE^{\sim}$ are skew- binormal operator in M.

Theorem 3.9. IF E is a m-symmetric operator then E^{-1} is also a skew-binormal operator in Minkowski space M.

Proof:

By the definition of m-symmetric, $E^{\sim} = E$

To Prove E^{-1} is skew-binormal in M, it is enough to check if it is a m-symmetric.

Now, $(E^{-1})^{\sim} = (E^{\sim})^{-1} = E^{-1}$

Therefore, E^{-1} i s m-symmetric operator.

Hence it is proved that E^{-1} is skew-binormal operator in Minkowski space M.

Proof verification:

Assume that E^{-1} is m-symmetrix operator.

Therefore $(E^{-1})^{\sim} = E^{-1}$

To prove that E^{-1} is skew-binormal operator.

Consider,

$(E^{\sim}EEE^{\sim})E = (E^{-1})^{\sim}E^{-1}E^{-1}(E^{-1})^{\sim}E^{-1}$

$= E^{-1}E^{-1}E^{-1}E^{-1}E^{-1}$

$= E^{-5}$ .... (1)

$E(EE^{\sim}E^{\sim}E) = E^{-1}E^{-1}(E^{-1})^{\sim}(E^{-1})^{\sim}E^{-1})$

$= E^{-1}E^{-1}E^{-1}E^{-1}E^{-1}$

$= E^{-5}$... (1)

From (1) and (2),

E^{-1} is skew-binormal operator in Minkowski space M.

Theorem 3.10. Consider an operator E which is a m-symmetric operator on a minkowski space M, then F ~EF is skew-binormal in Minkowski space M.

Proof:

Since E is m-symmetric operator, E~ = E

Consider,

(F~EF)~ = (F~E~F~ ~) = F~EF

Therefore, F~EF is m-symmetric operator.

By theorem (3.2), we have the result that if F~EF is m-symmetric operator in M, then it is skew-binormal in M.

Assume F~EF is m-symmetric,

Consider,

[(F ~EF)~(F ~EF)(F ~EF)(F ~EF)~]F ~EF

= (F ~E~F)(F ~EF)(F ~EF)(F ~E~F)(F ~EF)

= (F ~EF)~)^5 .... (1)

(F ~EF)[(F ~EF)(F ~EF))~(F ~EF)~(F ~EF)]

= (F ~EF)(F ~EF)(F ~EF)(F ~EF)(F ~EF)

= (F ~EF)~)^5 .... (2)

From (1) and (2), (F~EF) is skew-binormal operator in Minkowski space M .

4. p-skew-bi-normal opertor in Minkowki space

Theorem 4.1. If E is a normal operator in minkowski space M. Then E is said to be P-skew-bi-normal opertor in minkowspace M.

Proof:

Suppose E is a normal operator in M.

We need to prove that

$$(E^{\sim}EEE^{\sim})(E^{\sim})^{P} = (E^{\sim})^{P}(E^{\sim}EEE^{\sim})$$

$$\begin{aligned}
(E^{\sim}EEE^{\sim})(E^{\sim})^{P} &= E^{\sim}EEE^{\sim}(E^{\sim})^{P} \\
&= E\,E^{\sim}E^{\sim}\,E\,(E^{\sim})^{P} \\
&= EE^{\sim}E^{\sim}(E^{\sim})^{P}E \\
&= EE^{\sim}(E^{\sim})^{P}E^{\sim}E \\
&= E(E^{\sim})^{P}E^{\sim}E^{\sim}E \\
&= (E^{\sim})^{P}(EE^{\sim}E^{\sim}E)
\end{aligned}$$

$$(E^{\sim}EEE^{\sim})(E^{\sim})^{P} = (E^{\sim})^{P}(EE^{\sim}E^{\sim}E)$$

Hence, E is P-skew-bi-normal operator in Minkowski space M.

Theorem 4.2. If E and F are normal, P-skew-bi-normal operator in M, let E Commute with F then (EF) is P-skew-bi-normal operator M.

Proof:
Since E and F are P-skew-bi-normal operator, we have

$$(E^{\sim}EEE^{\sim})(E^{\sim})^{P} = (E^{\sim})^{P}(EE^{\sim}E^{\sim}E) \quad \text{and} \quad (F^{\sim}FFF^{\sim})(F^{\sim})^{P} = (F^{\sim})^{P}(FF^{\sim}F^{\sim}E)$$

$$\begin{aligned}
[(EF)^{\sim}(EF)(EF)(EF)^{\sim}][(EF)^{\sim P}] &= [(F^{\sim}E^{\sim})(EF)(EF)(F^{\sim}E^{\sim})][(F^{\sim})^{P}(E^{\sim P})] \\
&= [F^{\sim}E^{\sim}EFEFF^{\sim}E^{\sim}][(F^{\sim})^{P}E^{\sim P}] \\
&= [EF^{\sim}E^{\sim}FEF^{\sim}E^{\sim}F][(F^{\sim})^{P}E^{\sim P}] \\
&= [EF^{\sim}FE^{\sim}F^{\sim}EE^{\sim}F][(F^{\sim})^{P}E^{\sim P}] \\
&= [EFF^{\sim}E^{\sim}F^{\sim}E^{\sim}EF][(F^{\sim})^{P}E^{\sim P}] \\
&= EFF^{\sim}E^{\sim}F^{\sim}E(F^{\sim})^{P}FE^{\sim P} \\
&= EFF^{\sim}E^{\sim}F^{\sim}E^{\sim}E(F^{\sim})^{P}(E^{\sim})^{P}F \\
&= EFF^{\sim}E^{\sim}F^{\sim}E^{\sim}(F^{\sim})^{P}E(E^{\sim})^{P}F \\
&= EFF^{\sim}E^{\sim}F^{\sim}E^{\sim}(F^{\sim})^{P}(E^{\sim})^{P}EF \\
&= EFF^{\sim}E^{\sim}F^{\sim}(F^{\sim})^{P}E^{\sim}(E^{\sim})^{P}EF \\
&= EFF^{\sim}E^{\sim}F^{\sim}(F^{\sim})^{P}(E^{\sim})^{P}E^{\sim}EF
\end{aligned}$$

$$= EFF\sim E\sim (F\sim)^P F\sim(E\sim)^P E\sim EF$$

$$= EFF\sim E\sim (F\sim)^P (E\sim)^P F\sim E\sim EF$$

$$= EFF\sim (F\sim)^P E\sim (E\sim)^P F\sim E\sim EF$$

$$= EFF\sim (F\sim)^P (E\sim)^P E\sim F\sim E\sim EF$$

$$= EF(F\sim)^P F\sim (E\sim)^P E\sim F\sim E\sim EF$$

$$= E(F\sim)^P (E\sim)^P F\sim E\sim F\sim E\sim EF$$

$$= E(F\sim)^P (E\sim)^P FF\sim E\sim F\sim E\sim EF$$

$$= (F\sim)^P E (E\sim)^P FF\sim E\sim F\sim E\sim EF$$

$$= (F\sim)^P (E\sim)^P EFF\sim E\sim F\sim E\sim EF$$

$$= (EF)\sim)^P (EF)(F\sim E)\sim (F\sim E\sim)EF$$

$$= [(F\sim)^P (E\sim)^P].[(EF)(F\sim E\sim)(F\sim E\sim)(EF)]$$

$$= [((EF)\sim)^P].[(EF)(F\sim E\sim)(F\sim E\sim)(EF)]$$

$$[(EF)\sim(EF)(EF)(EF)\sim][(EF)\sim^P] = [((EF)\sim)^P].[(EF)(F\sim E\sim)(F\sim E\sim)(EF)]$$

Hence (EF) is P-skew-bi-normal operator M .

Theorem 4.3. If E^{-1} exists and E is a P-skew-bi-normal operator in Minkowski spac M, then E^{-1} is P-skew-bi-normal operator in Minkowski space M.

Proof:
Since E is p-skew-bi-normal operator in M,

We have $(E\sim EEE\sim)(E\sim)^P = (E\sim)^P(EE\sim E\sim E)$

Let

$$[(E^{-1})^{\sim}) E^{-1}E^{-1}(E^{-1})\sim][((E^{-1})\sim)^P$$

$$= [(E^{\sim -1})(E^{-1}E^{-1}(E^{-1})\sim][(E^{\sim P})^{-1}$$

$$= [(EE\sim)^{-1}(E\sim E)^{-1}]((E\sim)^P)^{-1}$$

$$= [(E\sim E)^{-1} (EE\sim)^{-1}] ((E\sim)^P)^{-1}$$

$$= [(E\sim)^P(E\sim EEE\sim)]^{-1}$$

$= [(EE\sim E\sim E)(E\sim)^P]^{-1}$,[Since E is a p-skew-bi-normal].

$= [(E\sim EEE\sim)(E\sim)^{P}]^{-1}$, [Since E is binormal]

$= (((E\sim)^{P}))^{-1}[(E\sim EEE\sim)]^{-1}$

$= (((E\sim)^{P}))^{-1}[(E\sim E)^{-1}(EE\sim)^{-1}]$

$= (((E\sim)^{P}))^{-1}[E^{-1}(E\sim)^{-1}(E\sim)^{-1}E^{-1}]$

$= \quad (((E^{-1})\sim))^{P} \quad [E^{-1}(E^{-1})\sim(E^{-1})\sim E^{-1}] \qquad [(E^{-1}\sim)E^{-1}E^{-1}(E^{-1})\sim]((E^{-1})\sim)^{P}=(((E^{-1})\sim))^{P}$ $[E^{-1}(E^{-1})\sim(E^{-1})\sim E^{-1}]$

Hence, E^{-1} is p-skew-bi-normal operator in Minkowski space M.

Theorem 4.4. Let E is p-skew-bi-normal operator in Minkowski space M. Then if F is unitary equivalent to E, then F is p-skew-bi-normal operator in Minkowski space M.

Proof:

Since F is unitary equivalent to E, we have $F = UEU\sim$, Let E be skew-bi-normal operator. We have $(E\sim EEE\sim)(E\sim)^{P}=(E\sim)^{P}(EE\sim E\sim E)$

$$(F\sim FFEF\sim)(F\sim)^{P} = [(UEU\sim)\sim(UEU\sim)(UEU\sim)(UEU\sim)\sim][((UEU\sim)\sim)^{P}]$$

$$=[(UE\sim U\sim)(UEU\sim)(UEU\sim)(UE\sim U\sim)](U(E\sim)^{P}U\sim)$$

$$= U[(E\sim EEE\sim)(E\sim)^{P}].(U\sim)\text{[Since E isP- Skew-bi-normal operator]},$$

$$=U[(E\sim)^{P}(EE\sim E\sim E)](U\sim)$$

$$= (U(E\sim)^{P}U\sim)[(UEU\sim)(UE\sim U\sim)(UE\sim U\sim)(UEU\sim)]$$

$$= (F\sim)^{P}(FF\sim F\sim F).$$

Hence, F is P- Skew-bi-normal operator. In Minkowski space M.

Theorem 4.5. If E is a m-symmetric and p- Skew-bi-normal operator in Minkowski space M, then $E\sim$ is P- Skew-bi-normal operator in Minkowski space M.

Proof:

Since E is P- Skew-bi-normal operator in M ,

we have $(E\sim EEE\sim)(E\sim)^{P} = (E\sim)^{P}(EE\sim E\sim E)$.

Replace E by $E\sim$ [By definition of m-symmetric]

Then we have

$$[(E\sim)\sim E\sim E\sim(E\sim)\sim]((E\sim)\sim)^{P} = (E\sim EEE\sim)(E\sim)^{P}\text{ , Since E is m-symmetri}$$

$$=(E\sim)^{P}(E\sim EEE\sim)\text{ ,[Since E is P- Skew-bi-normal]}$$

$$= ((E\sim)\sim)^{P}[E\sim(E\sim)\sim(E\sim)\sim E\sim],$$

[Since E is m-symmetrix]

Hence, E~ is P- Skew-bi-normal operator in Minkowski space M .

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